

State the Squeeze Theorem.

SCORE: ____ / 10 PTS

IF $f(x) \leq g(x) \leq h(x)$ ON AN OPEN INTERVAL AROUND $x=a$
(EXCEPT POSSIBLY AT $x=a$)

AND $\lim_{x \rightarrow a} f(x) = \lim_{x \rightarrow a} h(x) = L$

THEN $\lim_{x \rightarrow a} g(x) = L$

Let $f(x) = 7 - 3x^2 - x^3$.

SCORE: ____ / 30 PTS

- [a] Find $f'(x)$. No credit will be given for using differentiation shortcuts from chapter 3.

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{7 - 3(x+h)^2 - (x+h)^3 - (7 - 3x^2 - x^3)}{h} \\ &= \lim_{h \rightarrow 0} \frac{7 - 3(x^2 + 2xh + h^2) - (x^3 + 3x^2h + 3xh^2 + h^3) - 7 + 3x^2 + x^3}{h} \\ &= \lim_{h \rightarrow 0} \frac{-6xh - 3h^2 - 3x^2h - 3xh^2 - h^3}{h} \\ &= \lim_{h \rightarrow 0} (-6x - 3h - 3x^2 - 3xh - h^2) \\ &= -6x - 3x^2 \end{aligned}$$

- [b] Find the equation of the tangent to the graph of f at the point where $x = -1$.

$$\begin{aligned} f(-1) &= 7 - 3 + 1 = 5 & f'(-1) &= 6 - 3 = 3 \\ y - 5 &= 3(x + 1) \end{aligned}$$

At time t hours, the position of an object moving in a straight line is $p(t) = \frac{38+5t}{4+t}$ meters.

SCORE: ____ / 25 PTS

- [a] What is the average velocity of the object from time $t = 2$ to time $t = 5$? Give the units of your answer.

$$\frac{p(5) - p(2)}{5 - 2} = \frac{\frac{63}{9} - \frac{48}{6}}{3} = \frac{7 - 8}{3} = -\frac{1}{3} \text{ m/hr}$$

- [b] What is the instantaneous velocity of the object at time $t = 2$? Give the units of your answer.

$$\lim_{b \rightarrow 2} \frac{\frac{38+5b}{4+b} - 8}{b-2}$$

$$= \lim_{b \rightarrow 2} \frac{38+5b-8(4+b)}{(b-2)(4+b)}$$

$$= \lim_{b \rightarrow 2} \frac{b-3b-3}{(b-2)(4+b)}$$

$$= \frac{-3}{6}$$

$$= -\frac{1}{2} \text{ m/hr}$$

$$\text{OR } \lim_{h \rightarrow 0} \frac{\frac{38+5(2+h)}{4+(2+h)} - 8}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\frac{48+5h}{6+h} - 8}{h}$$

$$= \lim_{h \rightarrow 0} \frac{48+5h-8(6+h)}{h(6+h)}$$

$$= \lim_{h \rightarrow 0} \frac{-3h}{h(6+h)}$$

$$= \frac{-3}{6}$$

$$= -\frac{1}{2} \text{ m/hr}$$

Use the Squeeze Theorem to prove that $\lim_{x \rightarrow \infty} \frac{\sin x}{x} = 0$.

SCORE: ____ / 10 PTS

$$-1 \leq \sin x \leq 1$$

$$-\frac{1}{x} \leq \frac{\sin x}{x} \leq \frac{1}{x} \quad \text{AS } x \rightarrow \infty$$

$$\lim_{x \rightarrow \infty} -\frac{1}{x} = 0 = \lim_{x \rightarrow \infty} \frac{1}{x}$$

BY SQUEEZE THEOREM, $\lim_{x \rightarrow \infty} \frac{\sin x}{x} = 0$

The monthly heating/cooling cost for the classrooms depends on the temperature at which they are kept.

SCORE: ____ / 15 PTS

Let $c = g(t)$, where c is the monthly cost (in hundreds of dollars), and t is the temperature (in $^{\circ}F$).

[a] What are the units of $g'(t)$?

HUNDREDS OF $\$/^{\circ}F$

[b] What does $g'(62) = -5$ mean? Give the units for all numbers in your answer.

IF THE ROOMS ARE KEPT AT $62^{\circ}F$,
THEN FOR EACH $1^{\circ}F$ HIGHER THE ROOMS ARE KEPT,
\$500 WOULD BE SAVED EACH MONTH

[c] Is there a value of t_0 for which you would expect $g'(t_0) > 0$? Why or why not?

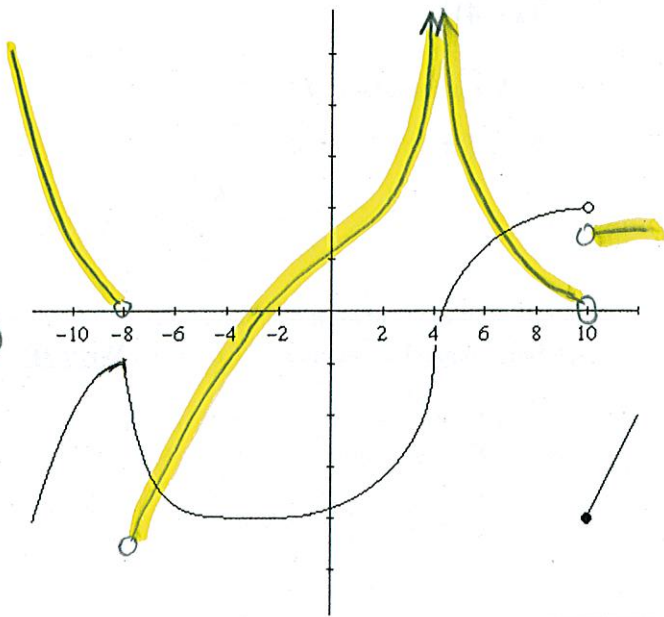
YES. IF THE ROOMS ARE KEPT EXTREMELY HOT (EG. $> 100^{\circ}F$)
RAISING THE TEMPERATURE WOULD COST MORE

The graph of $f(x)$ is shown on the right.

SCORE: ___ / 20 POINTS

- [a] List all x - values for which $f'(x)$ does not exist.
For each x - value, give a **brief** reason why not.

$x = -8$ (CUSP)
 $x = 4$ (VERTICAL T.L.)
 $x = 10$ (DISCONTINUITY)



- [b] Sketch a graph of $f'(x)$ on the same axes.

State the formal definition of "horizontal asymptote".

SCORE: ____ / 5 PTS

f HAS A HORIZONTAL ASYMPTOTE AT $y=b$ IFF

$$\lim_{x \rightarrow \infty} f(x) = b \text{ or } \lim_{x \rightarrow -\infty} f(x) = b$$

Find the equation(s) of all horizontal asymptote(s) of $f(x) = \frac{9-6e^{-x}}{13e^{-x}+4}$.

SCORE: ____ / 15 PTS

$$\lim_{x \rightarrow \infty} \frac{9-6e^{-x}}{13e^{-x}+4} = \frac{9-0}{0+4} = \frac{9}{4}$$

$$\lim_{x \rightarrow -\infty} \frac{9-6e^{-x}}{13e^{-x}+4} = \lim_{x \rightarrow -\infty} \frac{9e^x - 6}{13+4e^x} = \frac{0-6}{13+0} = -\frac{6}{13}$$

$\frac{-\infty}{\infty}$

$$y = \frac{9}{4} \text{ AND } y = -\frac{6}{13}$$

Let $f(x) = \frac{x^2 - 1}{x(x+4)^2}$.

SCORE: ____ / 20 PTS

- [a] Find all discontinuities of f .

$$x(x+4)^2 = 0$$

$$x = 0, -4$$

- [b] Find the limit of f at each discontinuity.

Each limit should be a number, ∞ or $-\infty$. Write DNE only if the other possibilities do not apply.

$$\lim_{x \rightarrow 0^+} \frac{x^2 - 1}{x(x+4)^2} = -\infty$$

$$\frac{-1}{0^+(16)}$$

$$\lim_{x \rightarrow 0^-} \frac{x^2 - 1}{x(x+4)^2} = \infty$$

$$\frac{-1}{0^-(16)}$$

$$\lim_{x \rightarrow 0} \frac{x^2 - 1}{x(x+4)^2} = \text{DNE}$$

$$\lim_{x \rightarrow -4^+} \frac{x^2 - 1}{x(x+4)^2} = -\infty$$

$$\frac{15}{-4(0^+)}$$

$$\lim_{x \rightarrow -4^-} \frac{x^2 - 1}{x(x+4)^2} = -\infty$$

$$\frac{15}{-4(0^-)}$$

$$\lim_{x \rightarrow -4} \frac{x^2 - 1}{x(x+4)^2} = -\infty$$